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Alignment Requirements for the Run IIb Silicon Tracker

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Abstract

This note derives explicitly the required alignment tolerances for the Run IIb silicon tracker. Assembly, survey and *in situ* alignment constraints are discussed.

1 Introduction

The Run IIb silicon tracker consists of six layers of silicon arranged in a barrel geometry. All silicon is single sided, with axial readout in all layers. The outer 4 layers also contain stereo readout with stereo angles of roughly 2.48° for $|z| < 200mm$ and 1.24° for $200 < |z| < 600mm$. This note goes through in detail the propagation of sensor alignment errors into errors in single-hit resolution in the axial sensors. Based on expected sensor resolutions we derive the translational and rotational tolerances required. Finally we discuss these alignment issues in the context of the STT trigger performance and ability to produce physics results as rapidly as possible after installation of the new tracker.

2 Derivation of resolution smearing

For a given sensor there are six degrees of freedom for the location and orientation of the best-fit plane of the sensor, relative to ideal, plus deviations from a perfect plane. For the following discussion we define the coordinate system so that the Z axis is along the axial strip direction, X is perpendicular to the strips and Y is out of the plane of the sensors. In terms of detector coordinates, $X \approx \phi,\ Y \approx R,\ Z = Z$. The single hit resolution for an axial sensor is simply given by the error in X in these coordinates. Two of the six possible errors are trivial cases: errors in Z have no effect on the measurement while errors in X contribute directly.

The effects of the four non-trivial error sources are derived below. To derive the average effect of an error we assume uniform track density in ϕ and Z over the silicon region of interest. Note that the coordinate system origin is the centroid of the readout segment of width W and length L.

2.1 Radial offset of the sensor

For a radial offset ΔY of a sensor, the error in hit position is given by

$$\delta X(X,Z) = X\frac{\Delta Y}{R}$$

The average error, $< \delta X >= 0$. The RMS error is obtained by integration over ϕ and Z. We have transformed the integration over ϕ into an integration over X using $d\phi = \frac{R}{X^2+R^2}dX$, with the result

$$< (\delta X)^2 >= \frac{\int_{-L/2}^{L/2}\int_{-W/2}^{W/2}(\frac{X\Delta Y}{R})^2\frac{R}{X^2+R^2}dXdZ}{\int_{-L/2}^{L/2}\int_{-W/2}^{W/2}\frac{R}{X^2+R^2}dXdZ}$$

Carrying out the integration we find

$$\sqrt{< (\delta X)^2 >} = \Delta Y\sqrt{\frac{W}{2RC} - 1}$$

where $C \equiv \int_0^{W/2}\frac{R}{X^2+R^2}dX = tan^{-1}(W/2R)$.

2.2 Rotation about a circumferential axis, θ_X

For a rotation about the X axis, θ_X, the error in hit position is given by

$$\delta X(X, Z) = XZ\frac{sin\theta_X}{R}$$

The average error, $< \delta X >= 0$. The RMS error is given by

$$< (\delta X)^2 >= \frac{\int_{-L/2}^{L/2}\int_{-W/2}^{W/2}X^2Z^2(\frac{sin\theta_X}{R})^2\frac{R}{X^2+R^2}dXdZ}{\int_{-L/2}^{L/2}\int_{-W/2}^{W/2}\frac{R}{X^2+R^2}dXdZ}$$

Carrying out the integration we find

$$\sqrt{< (\delta X)^2 >} = Lsin\theta_X\sqrt{\frac{W}{24RC} - \frac{1}{12}}$$

2.3 Rotation about a radial axis, θ_Y

For a rotation about the Y axis, θ_Y, the error in hit position is given by

$$\delta X(X, Z) = Zsin\theta_Y$$

The average error, $< \delta X >= 0$. The RMS error is given by

$$< (\delta X)^2 >= \frac{\int_{-L/2}^{L/2}\int_{-W/2}^{W/2}Z^2sin^2\theta_Y\frac{R}{X^2+R^2}dXdZ}{\int_{-L/2}^{L/2}\int_{-W/2}^{W/2}\frac{R}{X^2+R^2}dXdZ}$$

Carrying out the integration we find

$$\sqrt{< (\delta X)^2 >} = \frac{L}{\sqrt{12}}sin\theta_Y$$

2.4 Rotation about an axis parallel to beam line, θ_Z

For a rotation about the Z axis, θ_Z, the error in hit position is given by

$$\delta X(X,Z) = X^2 \frac{sin\theta_Z}{R}$$

Note that in this case the average error is not zero, resulting in a systematic error in ϕ in addition to the resolution smearing.

$$< \delta X >= \frac{\int_{-L/2}^{L/2} \int_{-W/2}^{W/2} X^2 \frac{sin\theta_Z}{R} \frac{R}{X^2+R^2} dX dZ}{\int_{-L/2}^{L/2} \int_{-W/2}^{W/2} \frac{R}{X^2+R^2} dX dZ}$$

which results in

$$< \delta X >= sin\theta_Z \frac{W - 2RC}{2C}$$

The RMS error is given by

$$< (\delta X)^2 >= \frac{\int_{-L/2}^{L/2} \int_{-W/2}^{W/2} X^4 (\frac{sin\theta_Z}{R})^2 \frac{R}{X^2+R^2} dX dZ}{\int_{-L/2}^{L/2} \int_{-W/2}^{W/2} \frac{R}{X^2+R^2} dX dZ}$$

Carrying out the integration we find

$$\sqrt{< (\delta X)^2 >} = sin\theta_Z \sqrt{\frac{W^3}{24RC} - \frac{WR}{2C} + R^2}$$

2.5 Sensor flatness

The incorporation of an arbitrary non-planar sensor geometry is untenable. The predominant free-state shape of the sensors is a "tenting" up at the center. This is dominantly present along the length of the sensors. If one approximates this shape as a linear (V-shaped) distortion along Z only, then the Z dependence of the error in hit position changes from $Zsin\theta_X$ to $(|Z| - L/4)sin\theta_X$ and the result obtained Section 2.2 for $\sqrt{< (\delta X)^2 >}$ is reduced by a factor of 2. For tenting across the width of the sensor ("ϕ") one expects a similar reduction in RMS error compared to a pure rotation θ_Z from Section 2.4, but in this case there is no systematic error, i.e. $< \delta X >= 0$.

For the Run IIa tracker the sensor flatness was not as well constrained by the support structure as for the Run IIb tracker, with flatness values having an RMS of $\approx 40\mu$ for the ladders used in the innermost layers and maximum values in excess of 100μ. In the Run IIb tracker we expect to have the flatness controlled a factor of 2 to 3 better so that this contribution to the resolution in negligible [1].

2.6 Summary of resolution smearing

Table 1 summarizes the results obtained in the previous subsections.

Error	$< \delta X >$	$\sqrt{(< \delta X >)^2}$
ΔX	ΔX	ΔX
ΔY	0	$\Delta Y \sqrt{\frac{W}{2RC} - 1}$
ΔZ	0	0
θ_X	0	$L sin\theta_X \sqrt{\frac{W}{24RC} - \frac{1}{12}}$
θ_Y	0	$\frac{L}{\sqrt{12}} sin\theta_Y$
θ_Z	$sin\theta_Z \frac{W-2RC}{2C}$	$sin\theta_Z \sqrt{\frac{W^3}{24RC} - \frac{WR}{2C} + R^2}$
$C \equiv \int_0^{W/2} \frac{R}{X^2+R^2} dX = tan^{-1}(W/2R)$		

Table 1: Formulas for average and RMS errors on X ($\approx \phi$) measurement for alignment errors in each of the six degrees of freedom. Displacements and rotations are all taken about the physical centroid of the readout unit. L and W are the length and width of the readout unit.

3 Numerical evaluation of tolerances

In order to numerically evaluate the required alignment tolerances from the expressions above we must decide on a criterion to use. As a standard we require that each source contribute less than 10% to the resolution when added in quadrature to the intrinsic detector resolution. For the inner layer

[1] The prototype staves built to date on non-ideal tooling have flatness values of $\simeq 30\mu$ over each readout segment. Based on the small sample size in hand, our estimate is that the RMS of the flatness can be kept below 20μ

which has 50μ pitch the intrinsic resolution is taken to be 8.8μ[2]. For the outer layers which have $58-60\mu$ pitch we use $10.2-10.6\mu$. This means that each of the RMS errors above should stay below 4.0μ for layer 0 and below 4.8μ for the outer layers.

The results based on these criteria are summarized in Table 2.

	Tolerances (μm)				
Layer	ΔX	ΔY	$L\ sin\theta_X$	$L\ sin\theta_Y$	$W\ sin\theta_Z$
L0A	4.0	20	68	14	51
L0B	4.0	27	93	14	70
L1A	4.6	25	88	16	66
L1B	4.6	28	99	16	74
L2A	4.8	24	83	17	62
L2B	4.8	31	106	17	79
L3A	4.8	38	131	17	98
L3B	4.8	44	152	17	113
L4A	4.8	51	175	17	131
L4B	4.8	57	196	17	146
L5A	4.8	63	219	17	164
L5B	4.8	69	240	17	179

Table 2: Misalignments required to produce 10% degradation of single hit resolution. Values are absolute deviation from zero, or one half of the allowable range. Lengths, L, are nominally the length of the readout unit (80-200mm). Widths, W, are the active widths of the sensors.

4 Discussion

Table 2 provides guidance as to the alignment tolerances required to reduce resolution smearing to a negligible level. The question then arises whether

[2]Based on test beam data for single-sided sensors from Run IIa[1]. Raw resolution was $10-11\mu$ in that data, and is more likely what one would expect in operations, but we will use the ideal resolution in the following analysis.

these are off-line alignment tolerances, pre-installation survey tolerances or actual assembly tolerances. This is discussed in detail below.

Furthermore it is important to clarify whether tolerances are "physics" tolerances, i.e. σ's, or engineering tolerances, i.e. maximum allowable deviations from ideal. Given that we have calculated the RMS resolution over the readout area, it is our feeling that the alignment tolerances should be viewed as engineering tolerances, with the values shown being the absolute value of the deviation from the mean, or one half of the range.

Finally, one must be careful about how the physical tolerance under consideration, e.g. an entire L0 assembly or stave, translates into alignment errors for each of the readout units involved. The formulas and numerical values provided are for translations and rotations about the geometrical center of the readout units.

4.1 The Silicon Track Trigger

The Silicon Track Trigger (STT) looks for displaced vertices by comparing the distance of closest approach of tracks to the beam spot using only the $R-\phi$ view. The resolution for the displaced vertex is limited by the knowledge of the beam spot, $\approx 30\mu$, rather than by the silicon single hit resolution. A detailed analysis was done by John Hobbs for the Run IIa tracker to evaluate the required alignment tolerances for each layer so that this intrinsic resolution is not compromised[2]. The results were $10\mu, 15\mu, 20\mu, 15\mu$ for the four layers of that device. This implies that the resolution figures given in Table 2 are a factor of 2-3 more stringent than required for the STT. It should be noted that these tolerances apply to the absolute position of the silicon relative the the Tevatron beam, not merely to an internal silicon coordinate system.

The STT does not have the Z measurement of the track available, except at the level of the readout segmentation - L=80mm for layer 0 and layer 1, L=100 or 200mm for layers 2-5. This means that, for the trigger, the assembly tolerances for θ_X and θ_Y are critical. Table 3 summarizes the limits imposed on the angular misalignments of the sensors to the beam line. The canonical $100\mu rad$ alignment tolerance reflects the requirement on θ_Y. In contrast the requirement on dR/dZ is an order of magnitude less stringent.

It is important to note that the relative clocking of the cylinder bulkheads translates almost directly into an error in this critical angle and therefore

	Tolerances ($mrad$)	
Layer	θ_X	θ_Y
L0A	2.19	.448
L0B	3.01	.448
L1A	2.45	.448
L1B	2.75	.448
L2A	1.31	.262
L2B	1.67	.262
L3A	2.06	.262
L3B	2.39	.262
L4A	2.76	.262
L4B	3.09	.262
L5A	3.46	.262
L5B	3.79	.262

Table 3: Assembly tolerances required by the STT. Angles are with respect to the Tevatron beam axis.

should be controlled at the $100\mu rad$ as well, or to 36μ over the bulkhead diameter. Any kink at Z=0 between the north and south halves of the detector also contributes directly to the θ_Y error, so the two barrels must be collinear to 60μ over 600mm. This is comparable in magnitude to the expected gravitational deflection of the loaded cylinders so some compensation at the Z=0 joint is likely to be required.

In principle all other alignment constants can be incorporated into the STT. In practice one would prefer to minimize the alignment constants required for the STT.

- ΔX
  It is not reasonable to expect that the absolute ϕ of each sensor will be accurate to $\pm 10\mu$[3] given the number of assembly errors that contribute to this (bearing placement on bulkheads, bulkhead alignment to cylinders, module alignment on staves, sensor alignment within modules).

[3]Here we are using the tightest requirement provided by J. Hobbs analysis from Run IIa

We should anticipate needing alignment constants for these offsets in the STT.

- ΔY
  This amounts to the radial location of the sensors. The required accuracy for the STT is $\pm 50\mu$[4]. For the staves we again suffer from multiple alignment tolerances making it questionable whether this accuracy will be achieved. In particular the angle θX of a full stave, L0/1 structure or L2-5 barrel generates errors ΔY that vary sensor to sensor. One would expect to need constants for these offsets in the STT, but initial alignment should be within a factor of 2 of what is nominally required by the STT.

- θ_Z
  The tolerance on this angle is not tight. We should easily meet not only the STT requirement, but also the tolerance required for off-line analysis.

4.2 Expected assembly and survey accuracies

We anticipate measuring the modules on the staves using CMMs with accuracy (in the plane) of $3 - 5\mu$ over the extent of the stave. We also anticipate locating the sensors to the stave mounting pins with an RMS accuracy of 5μ. The L0 and L1 modules will be installed and surveyed on their structures with similar accuracy. The resolution of the CMMs and locating of the modules in the vertical coordinate (radius in detector coordinates) is not as good, perhaps $10 - 15\mu$. Somewhat better measurement and placement accuracies are expected for locating the bearings on the bulkheads which the stave mounting pins engage.

The accuracy of the alignment of the four mounting bulkheads is not expected to be quite as good and it is not obvious that the beam deflection of the cylinders will be compensated. Therefore one expects deviations of order $50 - 75\mu$ from ideal for the mounting points of the staves.

[4]For L0 the STT requirement on ΔX is a factor of 2.5 looser than what was used to generate the numbers in Table 2. Assuming a 10μ accuracy requirement in layer 1 also leads to a 50μ requirement on ΔY

While we will make a best effort at perfect assembly, it is not unreasonable to anticipate assembly errors as large as 100μ over the length of a stave with accumulated survey accuracy of $15-30\mu$, perhaps even worse for the detector radial coordinate where the CMM's will be less accurate measuring the silicon optically.

Revisiting Table 2 we now look at the alignment and survey in the context of each error source. Here we look at only internal alignment within the silicon tracker. It is obvious that the six degrees of freedom determining the tracker location relative to the CFT and Tevatron beam will have to be done *in situ* with tracks to get sufficient accuracy for off-line tracking.

- ΔX
  For all layers we should not expect that we will meet this tolerance by construction. Assembly accuracy within staves (or within sensors in each $R-\phi$ sector of layer 0 and layer 1) should provide trace collinearity to $3-5\mu$. Surveys of the bearings on the bulkheads should have similar accuracy and the fit of the stave pins to the bulkhead bearings introduces another tolerance at this level. Two more similar tolerances are introduced by the survey relating the Z=0 and Z=600 bulkheads within a cylinder and the cylinder relative the overall detector assembly. This leads to a rough estimate of $8-12\mu$ for the survey accuracy of the X (ϕ) locations of the sensors. *In situ* alignment with tracks will be required to reduce this to the desired level.

- ΔY
  The radial assembly accuracy is not controlled at the same level as $\phi - Z$. Here we expect variations in supports, in glue thickness and in sensor flatness to contribute significantly. It is not anticipated that sensors will be treated as non-planar in the software so flatness is critical over the readout segments. This should be controlled at the 30μ level, thus contributing 15μ to the ΔY tolerance. Radial survey information will have an error formed by the measurement error for the optical survey of the sensors to the pins on the stave, $< 15\mu$, added with the several $3-5\mu$ errors described above that relate the pins to the bearings to the cylinders and finally to the detector assembly. This leads to an overall uncertainty of $20-25\mu$, just within the desired tolerances. From an assembly standpoint, in light of the combined bearing, bulkhead,

cylinder, full detector tolerances achieving the desired accuracy will be very challenging.

- ΔZ
  This is relevant only for the Z measurements from the stereo sensors. Assembly tolerances should suffice for adequate resolution at turn-on. Uncertainties in stave locations during cool-down are likely to be the most significant issue for this term.

- θ_X
  The pitch angle of the sensors on the staves is determined both by the shape of the tooling holding the modules during gluing and the shape of the stave core. The stave cores are unlikely to have large-scale variations that contribute significantly to this tolerance. Similarly we do not expect the layer 0 and layer 1 structure shapes to contribute significantly[5]. The tooling is specified with a flatness tolerance of 25μ. The initial prototypes demonstrated that this tolerance is likely to maximally contribute to the pitch angle of the sensors as the fixtures tend to make a vee or banana shape along their length. The next significant error is the locating tolerance of the pins to the center line of the stave. This could contribute up to 50μ over 600mm. The last significant contributions will come from bulkhead to bulkhead alignment and the overall angle of the cylinder in the detector frame, for which we will use a somewhat pessimistic figure of 50μ over 600mm for each. Summing these gives a total error $Lsin\theta_X = 28 - 38\mu$ for module lengths of 80-200mm. This is a factor of 2 or more better than required and can be significantly reduced using pre-installation survey data if desired.

- θ_Y
  During stave assembly (or module installation for layers 0 and 1) we expect to control this to $Lsin\theta_Y \approx 5\mu$ relative to the CMM coordinate system established off the stave pins (or reference fiducials for layers 0 and 1). We anticipate bearing placements on the bulkheads and pin to bearing fit to each contribute similar amounts, but with

[5] An exception to this would be a glue fillet between the sensor mounting surface and the stave end cap preventing the Z=0 end of the 10-10 module from seating properly. This was seen in prototyping and is being addressed in the stave assembly QC/QA procedures. Similar difficulties could arise at the ends of the layer 0 and layer 1 structures.

L=600mm rather than the sensor or readout unit length. The bulkhead to bulkhead alignment will be done to 50μ or better and similarly the cylinders will be aligned to the overall detector coordinates at that level or better. These errors sum to $Lsin\theta_Y = 13, 15, 25\mu$ for readout modules with L=80, 100, 200mm. In addition bulkhead clocking errors contribute. Assuming a clocking error of 36μ over the bulkhead diameter of 360mm, the additional error is $2 - 16\mu$ depending on the radius. For L0A to L1B we then get $Lsin\theta_Y < 14\mu$, within the desired tolerance. In the outer layers we will exceed the specified limits. The survey tolerances on the bulkheads will reduce their contributions from $50\mu/600mm$ to $10\mu/600mm$ or less, as well as reducing the clocking error to $10\mu/360mm$. This reduces the total error to $Lsin\theta_Y = 9 - 11\mu$, within the desired tolerances in all layers.

- θ_Z
  Sensors should be installed on staves and inner layer support structures with this angle determined to $< 15\mu$ over the sensor width. One may expect similar magnitude errors coming from uncertainties in the fiducial features used to set the CMM coordinate system during module installation. The bearing placement on the bulkheads contributes 5μ. The relative clocking of the bulkheads should be done to 36μ over the diameter of 360mm. This leads to a total error of $Wsin\theta_Z < 16\mu$, well below what is desired.

5 Summary

Based on the above analyses it is apparent that the estimated assembly tolerances can provide sufficient placement accuracy to obviate the need for sensor alignment constants for three of the six degrees of freedom, ΔZ, θ_X and θ_Z. Pre-installation survey data can be used to provide calibration constants sufficient for ΔY and θ_Y, leaving only ΔX requiring calibration with tracks. For these calibration data the staves (or $R - \phi$ segments in the inner layers) may be treated as planar rigid bodies so the total set of parameters required is only 216+12, where the 12 are the overall positions and angles of the two cylinders as installed. In principle by measuring several fiducials on each sensor with $3 - 5\mu$ accuracy we can extract the sensor centroid with sufficient precision, but a secondary alignment of sensors within staves (segments) may be required to fully realize the desired $4 - 5\mu$ accuracy for ΔX.

For the trigger the two angles θ_X and θ_Y are critical. The assembly tolerances are most important for meeting the requirements on θ_Y. In particular it is critical that the bulkhead clocking errors and and kink at Z=0 between the north and south barrels be kept below $100\mu rad$ to avoid degradation of the STT resolution.

References

[1] M. Roco, D0-Note 3405 (1998).

[2] Y. Gershtein et al., D0-Note 3849 (2001).